# PICTON HIGH SCHOOL

Creating Opportunities Achieving Success

## YEAR 12 MATHEMATICS

## TASK ONE

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| Due Date: Monday 12th February, 2018 | Assessment Name: Sighted Test |
| Mark: /35 | Weighting: 10 % |

**SYLLABUS OUTCOMES TO BE ASSESSED:**

H5 Applies appropriate techniques from the study of calculus, geometry, ~~probability, trigonometry and series~~ to solve problems
H6 – Uses the derivative to determine the features of the graph of the function.
H7 – Uses the features of a graph to deduce information about the derivative.
H9 – Communicates using mathematical language, notation, diagrams and graphs.

**Preliminary Outcomes Assessed**

P5-Understands the concept of a function and the relationship between a function and its graph
P6 – Relates the derivative of a function to the slope of its graph.
P7 – Determines the derivative of a function through routine application of the rules of differentiation.
P8 – Understands and uses the language and notation of Calculus.

**DIRECTIVES TO BE ASSESSED:**

Apply: To use relevant information and skills for a given situation
Uses: To seek or achieve an end by means of
Communicates: Chooses the correct way to give a mathematical answer
Relates: Shows a connection between
Determines: Finds a reasonable mathematical solution for
Understands: To grasp the idea or meaning of

**TASK DESCRIPTION:**

You have been given a number of questions from which a ONE hour examination will be created.
The examination will include FIVE multiple choice questions and THREE short answer, free response questions.
You will be required to prepare for this examination by completing the attached questions as a form of study/revision. The examination questions will be taken from the attached questions.

You will need to USE and APPLY your mathematical skills to show UNDERSTANDING of Calculus.
You will be required to COMMUNICATE USING language, notation, diagrams and graphs to DETERMINE the derivative and RELATE it to a function and its slope.

**You will be assessed on the following topics:**

Locus and the Parabola (Chapter 11-Preliminary)

Introduction to Calculus (Chapter 8 – Preliminary)
Geometrical Application of Calculus (Chapter 2 -HSC)

Note Preliminary outcomes can and will be examined in ALL HSC tasks.

ASSESSMENT CRITERIA – STUDENT CHECKLIST:
You will be assessed on your ability to:
Apply your mathematical skills to solve problems from the topics listed below.
Have you:
Studied the chapter - Locus and the Parabola (Chapter 11-Preliminary)?
Studied the chapter - Introduction to Calculus (Chapter 8 – Preliminary)?
Studied the chapter - Geometrical Application of Calculus (Chapter 2 -HSC)?

You will be required to prepare for this examination by completing the attached questions as a form of study/revision. The examination questions will be taken from the attached set of questions.

## SIGHTED QUESTIONS

1. Differentiate $f(x) = x^2 - 2x$ from first principles?

2.Evaluate $\lim_{x \to 4} \dfrac{x^2 - 3x - 4}{x - 4}$

3. What is the gradient of the line perpendicular to the line $2x + y + 3 = 0$?

4. What is the gradient of the tangent to the curve $y = x^2 - 6x$ when $x = 1$?

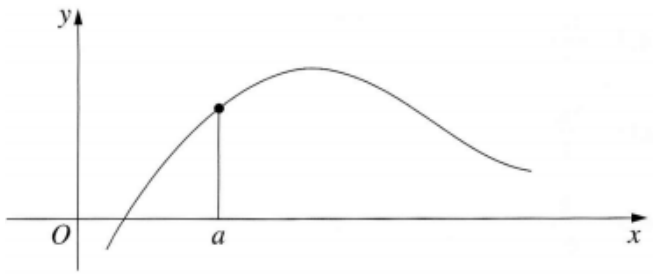
5. Graph $f(x) = 2x^3 - 3x^2$

6. What is the derivative of $(x - 5)^2(x + 3)^3$

7. What is the equation of the normal to the curve $y = x^2 - 4x$ at $(1, -3)$?

8. What values of $x$ is the curve $f(x) = x^3 + x^2$ concave down?

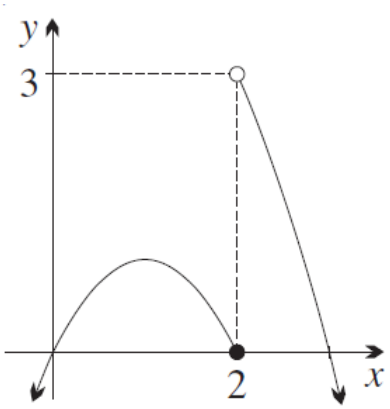
9. The diagram shows the graph of $y = f(x)$.




Look at where $f'(a) > 0$ or $f'(a) < 0$ and $f''(a) < 0$, $f''(a) < 0$ or $f''(a) = 0$

10. Which curve is differentiable at $x = 2$?

(A)

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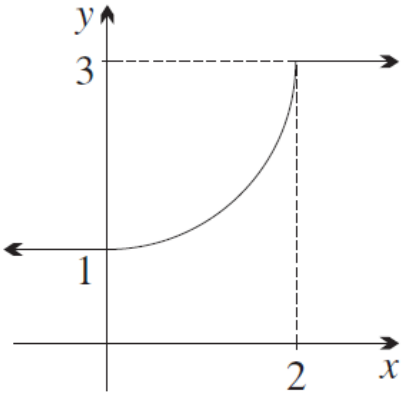


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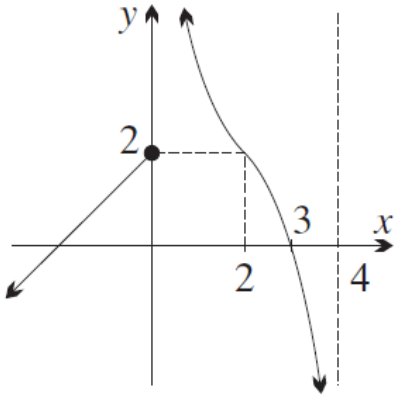


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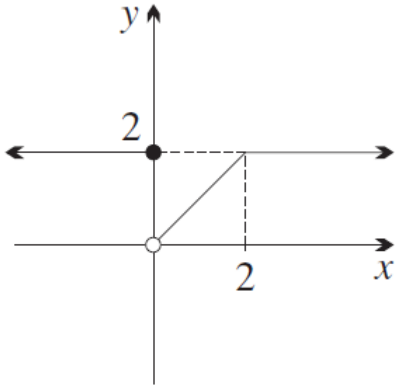



11. Evaluate $\lim_{x \to 3} \dfrac{x^2 - 2x - 3}{x - 3}$

12. For the parabola $x^2 = 16y$

(i) Find the coordinates of the focus.
(ii) Determine the equation of the directrix.

13. A point P $(x,y)$ moves so that it is equidistant from the point $L(-1,3)$ and the line $y = 0$. Show that the locus is a parabola and find its vertex.

14. Consider the equation of the curve: $y = x^2 - 5x$

i) Find the equation of the tangent to the curve at the point $P(2,-6)$.
ii) Find the equation of the normal to the curve at the point $P(2,-6)$.
iii) The tangent at $P$ meet the x-axis at $A$. Find the coordinates of $A$.
iv) The normal at $P$ meet the x-axis at $B$ respectively. Find the coordinates of $B$.
v) Find the length of the interval $AB$ and the area of the triangle $PAB$.

15. Differentiate from first principles $f(x) = x^2 - 4x + 8$

16. Find $\frac{dy}{dx}$ if :

(i) $y = (4x-2)(3x+6)$

(ii) $y = \frac{x^2-9}{x-3}$

17. A parabola has equation $(x-1)^2 = 8y$

Find the:

(i) coordinates of the vertex and focus.

(ii) the equation of the directrix.

Hence sketch the curve, showing the coordinates of the vertex, focus and directrix.

18. Find $\frac{dy}{dx}$ given $y = \frac{\sqrt{x-1}}{2x-3}$

19. Find the equation of the normal to the curve $f(x) = x^2 - 4x + 1$ at the point where $x = 1$.

20. Show that the equation of the normal to the curve $y = x^2$ at the point where $x = 2$ is given by $x + 4y - 18 = 0$ Hence:

(i) Find the coordinates of the vertex.

(ii) Find the coordinates of the focus.

(iii) Determine the equation of the directrix.

21. Find the equation of the tangent to the curve $y = x^3 + x^2 - x + 5$ at the point (1,6).

22. Find the derivative of $y = 2x^4 + 5x$.

23. Let $f(x) = x^3 - 3x^2 + kx + 8$, where $k$ is a constant. Find the values of $k$ for which $f(x)$ is an increasing function.

24. Differentiate each of the following:

i) $f(x) = (2x-7)^3$

ii) $g(x) = \frac{3x-5}{5x+2}$

iii) $h(x) = \sqrt{(3x-5)^5}$

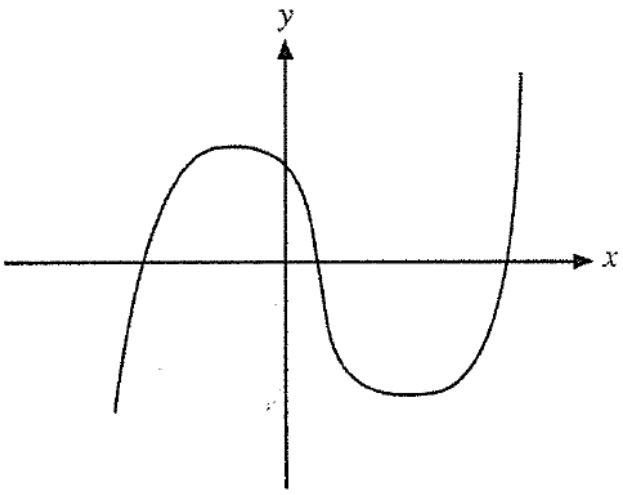
25. Find $\lim_{x \to 2} \frac{x^2+x-6}{x^2-4}$

26. The population *P* of fish in a certain lake was studied over time (*t*).
At the start the number of fish was 2500.

During the study, $\frac{dP}{dt} < 0$. What does this say about the number of fish during the study.

If at the same time, $\frac{d^2P}{dt^2} > 0$, sketch the graph of *P* against *t*.

27. Sketch the gradient (derivative) function of the following graph.




28. Consider the curve $y = 4x^3 - x^4$.

(i) Find the coordinates for the $x$ and $y$ intercepts.
(ii) Find the coordinates of the stationary points and determine their nature.
(iii) Find the coordinates of any points of inflexion.
(iv) Sketch the curve over the interval $-1 \leq x \leq 5$ showing all critical points.

29. Find the coordinates of the point at which the curve $y = x^3 + 1$ has a tangent with a gradient of 3.

30. Differentiate $f(x) = x^2 + 4x$ from first principals.

31. Consider the function $x^2 = -100y$.

(i) What are the co-ordinates of the focus?
(ii) What are the coordinates of the vertex?
(iii) What is the equation of the directrix?
(iv) Sketch the function showing all features.

32. Find the equation of the locus of point *P*(x,y) equidistant from *A*(3,6) and the line x = -7.

33. Consider the curve $y = x^3(2 - x)$.
(i) Find the coordinates of the stationary points and determine their nature.
(ii) Find the coordinates of any points of inflexion.
(iii) What is the minimum value of $y = x^3(2 - x)$. in the domain $-1 \leq x \leq 3$?

34. Find the equation of the tangent to the curve $y = x^3 + x^2 - x + 5$ at the point (1,6).

35. Differentiate $5 - 3x - 2x^2$ from first principles.

36. Differentiate each of the following

(i) $y = 10x^4$

(ii) $y = 4\sqrt{x}$

(iii) $y = (x - 7)(x + 10)$

(iv) $y = (7x^2 - 3)^4$

(v) $y = \dfrac{7x2}{(4x-6)3}$

37. The equation of a parabola is given by $x^2 - 4x - 2y + 8 = 0$. Find the:

(i) Vertex
(ii) Focus
(iii) Equation of the directrix of the parabola.
(iv) Equation of the focal chord that lies on the point (0,4) on the parabola.

38. Given two points *A* (3, -2) and *B* (-1, 7), find the equation of the locus *P* (*x*, *y*) if the gradient of *PA* is twice the gradient of *PB.*

39. Find the equation of the locus of a point that moves so that it is equidistant from the line $4x - 3y + 2 = 0$ and the line $3x + 4y - 7 = 0$

40. For the curve $y = x^3 - 27x - 5$, find values of $x$ for which $\frac{dy}{dx} = 0$.

41. Find the equation of the tangent to the curve $y = \frac{1}{x^3}$ at the point $(2, \frac{1}{8})$

42. The tangent at the point *P* on the curve $y = 4x^2 + 1$ is parallel to the *x*- axis. Find the coordinates of *P*.

43. Find the coordinates of point *Q*, where the tangent to the curve $y = 5x^2 - 3x$ is parallel to the line $7x - y + 3 = 0$

44. Find the coordinates of the centre and the length of the radius of the circle:

$x^2 - 4x + y^2 - 2y - 4 = 0$

45. Given two points *A* (2, -5) and *B* (-4, 3), find the equation of the circle with diameter *AB.*

46. Find the perpendicular distance from *P* (2, -5) to the line $5x + 12y - 2 = 0$ and hence find the equation of the circle with centre *P* and tangent $5x + 12y - 2 = 0$.

47. A function $f(x) = x^2 + 4x - 12$ has a tangent with a gradient of -6 at a point *P* on the curve.
Find the coordinates of the point *P*.

48. Find the coordinates of the centre and the length of the radius of the circle:

$x^2 + 8x + y^2 - 4y - 5 = 0$

49. The line with equation $x - 3y - 27 = 0$ meets the parabola $y^2 = 4x$ at two points.
Find the coordinates.

50. Find the equation of the parabola with vertex (1, 0) and focus at (1,4)

51. Find the equation of the parabola with a vertex (-2, 2) and focus (-4, -2).
Find its equation.

52. A function is given by $f(x) = \sqrt[4]{x}$. Evaluate $f'(16)$.

53. Find the gradient of the tangent to the curve $y = \frac{3}{2x^2}$ at the point $(1, 1\frac{1}{2})$.

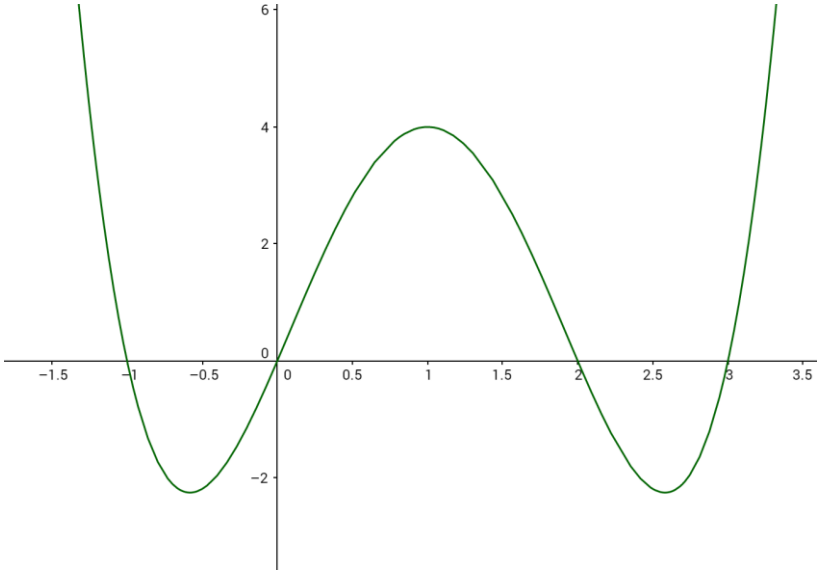
54. Find $\frac{dy}{dx}$ if $y = \left(x + \sqrt{x}\right)^2$

55. A function $f(x) = \frac{\sqrt{x}}{2}$ has a tangent at (4, 1). Find the gradient of the tangent.

56. The curve $y = ax^3 + bx^2 - x + 5$ has a point of inflexion at (1, -2)

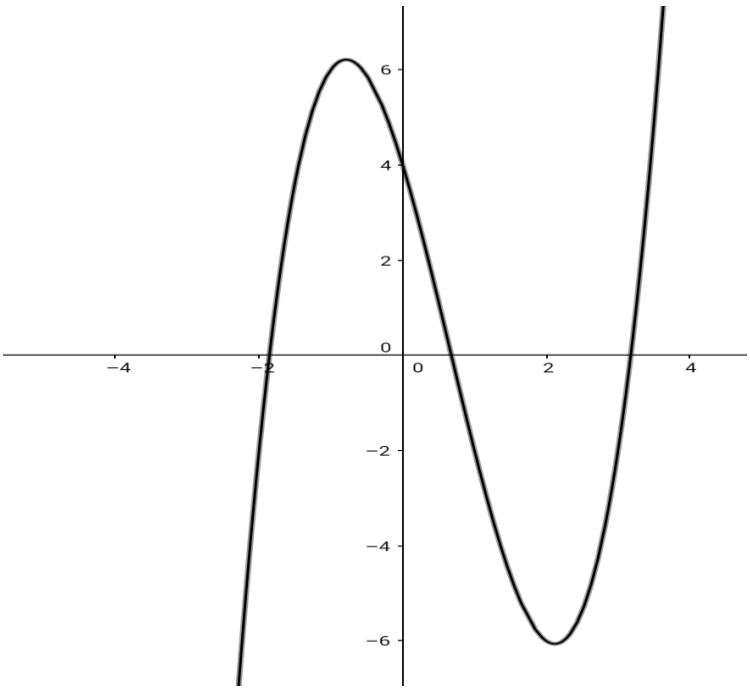
Find the values of a and b.

57. Draw the gradient function of the curve below:




58. Find all values of *x* for which the curve $f(x) = 2x^3 - 7x^2 - 5x + 4$ is concave downward.

59. Draw the gradient function of the curve below:




60. Find the point of inflexion on the curve $y = x^3 - 6x^2 + 5x + 9$