PICTON HIGH SCHOOL

Creating Opportunities Achieving Success

YEAR 12 MATHEMATICS

TASK ONE

Due Date: Wednesday 13th March, 2019 (Period 1)	Assessment Name: Sighted Test
Mark: /50	Weighting: 30 %

SYLLABUS OUTCOMES TO BE ASSESSED:
H1 Seeks to **apply** mathematical techniques to problems in a wide range of practical contexts
H2 Constructs arguments to **prove** and **justify** results
H5 **Applies** appropriate techniques from the study of calculus, geometry, ~~probability, trigonometry and series~~ to solve problems
H6 – **Uses** the derivative to determine the features of the graph of the function.
H7 – **Uses** the features of a graph to deduce information about the derivative.
H8 – **Uses** techniques of integration to calculate areas and volumes
H9 – **Communicates** using mathematical language, notation, diagrams and graphs.
Preliminary Outcomes Assessed
P5 - **Understands** the concept of a function and the relationship between a function and its graph
P6 – **Relates** the derivative of a function to the slope of its graph.
P7 – **Determines** the derivative of a function through routine application of the rules of differentiation.
P8 – **Understands** and uses the language and notation of Calculus.

DIRECTIVES TO BE ASSESSED:
Apply: To use relevant information and skills for a given situation
Prove: To provide logical evidence to support mathematical statement so support your mathematical claim.
Justify: To support an argument or conclusion.
Uses: To seek or achieve an end by means of
Communicates: Chooses the correct way to give a mathematical answer
Relates: Shows a connection between
Determines: Finds a reasonable mathematical solution for
Understands: To grasp the idea or meaning of

TASK DESCRIPTION:
You have been given a number of questions from which a 70 minute examination will be created.
The examination will include FIVE multiple choice questions and THREE short answer, free response questions.
You will be required to prepare for this examination by completing the attached questions as a form of study/revision. The examination questions will be taken from the attached questions.

You will need to USE and APPLY your mathematical skills to show UNDERSTANDING of the topics listed. You will be required to COMMUNICATE USING language, notation, diagrams and graphs to DETERMINE, PROVE or JUSTIFY the correct solution and RELATE it to the question.

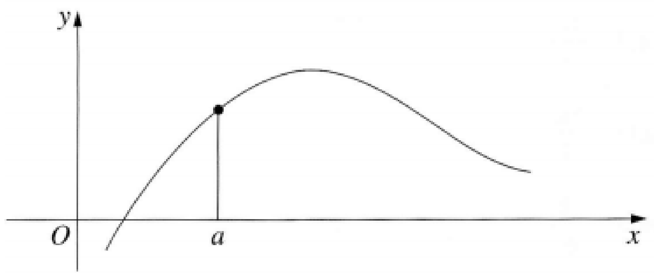
You will be assessed on the following topics:
Locus and the Parabola (Chapter 11 - Preliminary)
Introduction to Calculus (Chapter 8 - Preliminary)
Geometry 2 (Chapter 1 – HSC)
Geometrical Application of Calculus (Chapter 2 - HSC)
Integration (Chapter 3 - HSC)

Note Preliminary outcomes can and will be examined in ALL HSC tasks.

ASSESSMENT CRITERIA – STUDENT CHECKLIST:
You will be assessed on your ability to:
Apply your mathematical skills to solve problems from the topics listed below.
Have you:
Studied the chapter - Locus and the Parabola (Chapter 11-Preliminary)?
Studied the chapter - Introduction to Calculus (Chapter 8 – Preliminary)?
Studied the chapter – Geometry 2 (Chapter 1 -HSC)?
Studied the chapter - Geometrical Application of Calculus (Chapter 2 -HSC)?
Studied the chapter - Integration (Chapter 3 -HSC)?

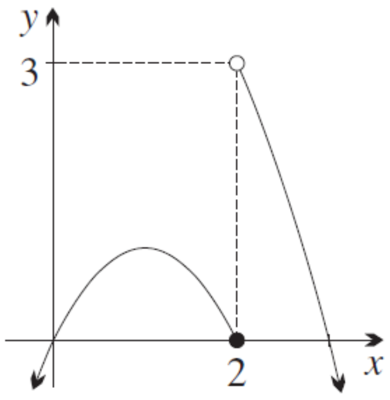
You will be required to prepare for this examination by completing the attached questions as a form of study/revision. The examination questions will be taken from the attached set of questions.

SIGHTED QUESTIONS

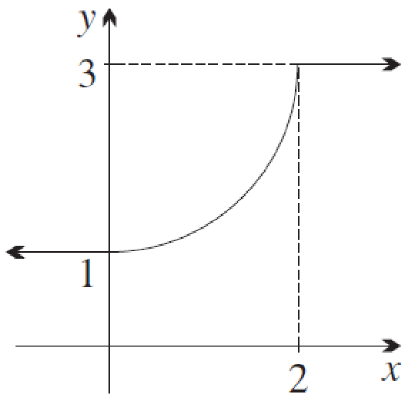
1. Differentiate $f(x) = x^2 - 2x$ from first principles?
2. Evaluate $\lim_{x \to 4} \frac{x^2 - 3x - 4}{x - 4}$
3. What is the gradient of the line perpendicular to the line $2x + y + 3 = 0$?
4. What is the gradient of the tangent to the curve $y = x^2 - 6x$ when $x = 1$?
5. Graph showing important features $f(x) = 2x^3 - 3x^2$
6. What is the derivative of $(x - 5)^2(x + 3)^3$
7. What is the equation of the normal to the curve $y = x^2 - 4x$ at $(1, -3)$?
8. What values of x is the curve $f(x) = x^3 + x^2$ concave down?
9. The diagram shows the graph of $y = f(x)$.  Look at where $f'(a) > 0$ or $f'(a) < 0$ and $f''(a) < 0$, $f''(a) < 0$ or $f''(a) = 0$ to assist you in determining the important features of this graph.
10. Evaluate $\lim_{x \to 3} \frac{x^2 - 2x - 3}{x - 3}$
11. For the parabola $x^2 = 16y$ (i) Find the coordinates of the focus. (ii) Determine the equation of the directrix.

12. Which curve is differentiable at $x = 2$?

(A)



(B)



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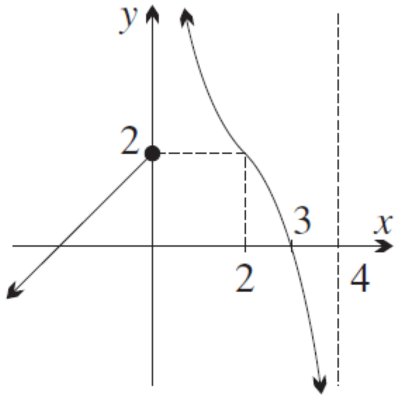

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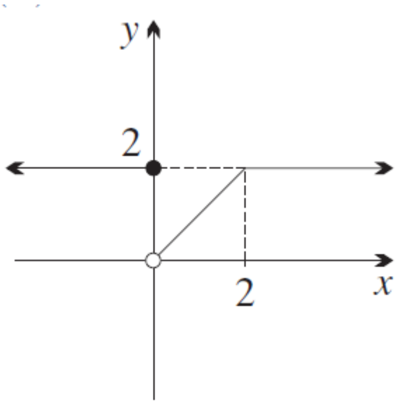


13. Using the Trapezoidal Rule, find an approximation for $\int_1^5 \frac{1}{x^2}dx$ using 4 subintervals

14. The gradient function of a curve $y = f(x)$ is given by $f'(x) = 4x - 5$. The curve passes through the point (2, 3). Find the equation of the curve.

15. A point P (x,y) moves so that it is equidistant from the point L(-1,3) and the line $y = 0$. Show that the locus is a parabola and find its vertex.

16. Consider the equation of the curve: $y = x^2 - 5x$

i) Find the equation of the tangent to the curve at the point $P(2,-6)$.
ii) Find the equation of the normal to the curve at the point $P(2,-6)$.
iii) The tangent at P meet the x-axis at A. Find the coordinates of A.
iv) The normal at P meet the x-axis at B respectively. Find the coordinates of B.
v) Find the length of the interval AB and the area of the triangle PAB.

17. Differentiate from first principles $f(x) = x^2 - 4x + 8$

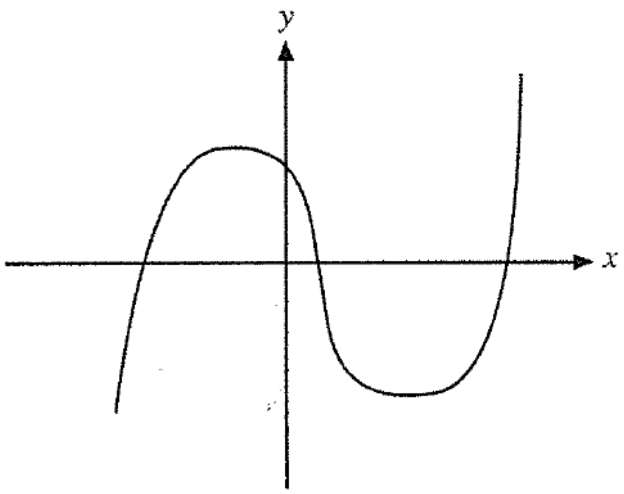
18. Find $\frac{dy}{dx}$ if : (i) $y = (4x-2)(3x+6)$ (ii) $y = \frac{x^2-9}{x-3}$
19. A parabola has equation $(x-1)^2 = 8y$ Find the: (i) coordinates of the vertex and focus. (ii) the equation of the directrix. Hence sketch the curve, showing the coordinates of the vertex, focus and directrix.
20. Find $\frac{dy}{dx}$ given $y = \frac{\sqrt{x-1}}{2x-3}$
21. Using Simpson's Rule, find an approximation for $\int_0^4 \sqrt{x}dx$ using 5 function values
22. Show that the equation of the normal to the curve $y = x^2$ at the point where $x = 2$ is given by $x + 4y - 18 = 0$ Hence: (i) Find the coordinates of the vertex. (ii) Find the coordinates of the focus. (iii) Determine the equation of the directrix.
23. Evaluate $\int_0^5 3x^2 dx$
24. Let $f(x) = x^3 - 3x^2 + kx + 8$, where k is a constant. Find the values of k for which $f(x)$ is an increasing function.
25. $\int x\sqrt{x}dx$
26. Find the primitive function of $\int (4+3x)^5 dx$
27. Find the coordinates of the centre and the length of the radius of the circle: $x^2 - 4x + y^2 - 2y - 4 = 0$
28. Find the equation of the locus of a point that moves so that it is equidistant from the line $4x - 3y + 2 = 0$ and the line $3x + 4y - 7 = 0$
29. A function is given by $f(x) = \sqrt[4]{x}$. Evaluate $f'(16)$.

30. The population P of fish in a certain lake was studied over time (t).
At the start the number of fish was 2500.

During the study, $\frac{dP}{dt} < 0$. What does this say about the number of fish during the study.

If at the same time, $\frac{d^2P}{dt^2} > 0$, sketch the graph of P against t.

31. Sketch the gradient (derivative) function of the following graph.



32. Consider the curve $y = 4x^3 - x^4$.

(i) Find the coordinates for the x and y intercepts.
(ii) Find the coordinates of the stationary points and determine their nature.
(iii) Find the coordinates of any points of inflexion.
(iv) Sketch the curve over the interval $-1 \leq x \leq 5$ showing all critical points.

33. Find the coordinates of the point at which the curve $y = x^3 + 1$ has a tangent with a gradient of 3.

34. For the curve $y = x^3 - 27x - 5$, find values of x for which $\frac{dy}{dx} = 0$.

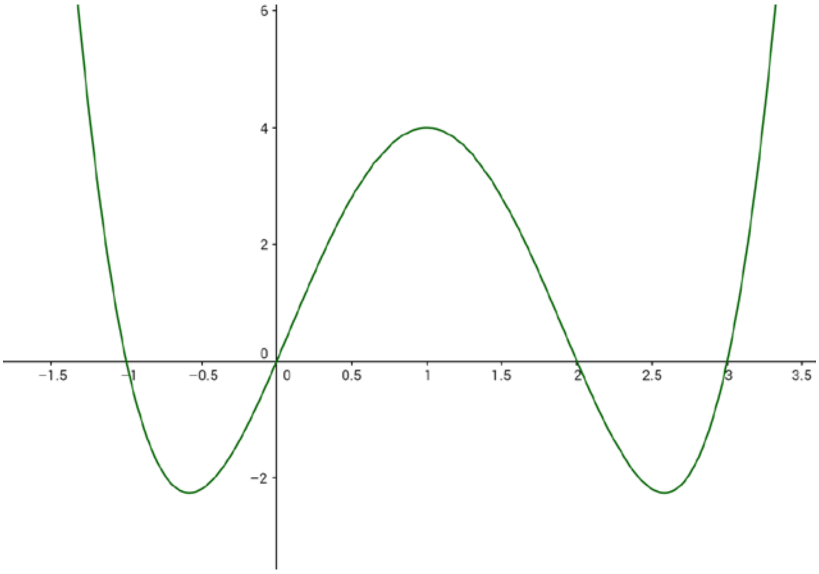
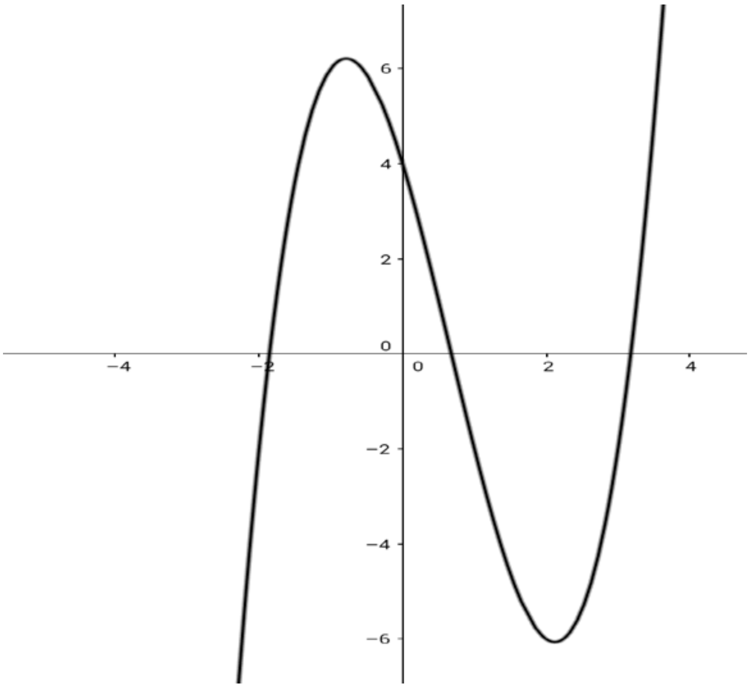
35. Consider the function $x^2 = -100y$.

(i) What are the co-ordinates of the focus?
(ii) What are the coordinates of the vertex?
(iii) What is the equation of the directrix?
(iv) Sketch the function showing all features.

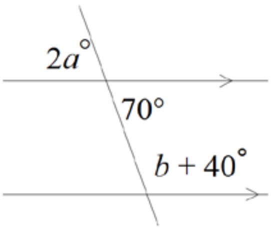
36. Find the equation of the locus of point $P(x,y)$ equidistant from $A(3,6)$ and the line $x = -7$.

37. Consider the curve $y = x^3(2 - x)$.
(i) Find the coordinates of the stationary points and determine their nature.
(ii) Find the coordinates of any points of inflexion.
(iii) What is the minimum value of $y = x^3(2 - x)$. in the domain $-1 \leq x \leq 3$?

38. The equation of a parabola is given by $x^2 - 4x - 2y + 8 = 0$. Find the: (i) Vertex (ii) Focus (iii) Equation of the directrix of the parabola. (iv) Equation of the focal chord that lies on the point (0,4) on the parabola.
39. Given two points *A* (3, -2) and *B* (-1, 7), find the equation of the locus *P* (*x*, *y*) if the gradient of *PA* is twice the gradient of *PB.*
40. Find the area enclosed between the curve $y = x^3$, the x-axis and the lines x = −2 and x = 2.
41. Find the area bounded by the curve $x = -y^2 - 5y - 6$ and the y-axis.
42. The tangent at the point *P* on the curve $y = 4x^2 + 1$ is parallel to the *x*- axis. Find the coordinates of *P*.
43. Find the coordinates of point *Q*, where the tangent to the curve $y = 5x^2 - 3x$ is parallel to the line $7x - y + 3 = 0$
44. Find the area enclosed by the curves $y = x^2$ and $x = y^2$
45. Given two points *A* (2, -5) and *B* (-4, 3), find the equation of the circle with diameter *AB.*
46. Find the perpendicular distance from *P* (2, -5) to the line $5x + 12y - 2 = 0$ and hence find the equation of the circle with centre *P* and tangent $5x + 12y - 2 = 0$.
47. A function $f(x) = x^2 + 4x - 12$ has a tangent with a gradient of -6 at a point *P* on the curve. Find the coordinates of the point *P*.
48. A function $f(x) = \frac{\sqrt{x}}{2}$ has a tangent at (4, 1). Find the gradient of the tangent.
49. The line with equation $x - 3y - 27 = 0$ meets the parabola $y^2 = 4x$ at two points. Find the coordinates.
50. Find the equation of the parabola with vertex (1, 0) and focus at (1,4)
51. Find $\frac{dy}{dx}$ if $y = \left(x + \sqrt{x}\right)^2$

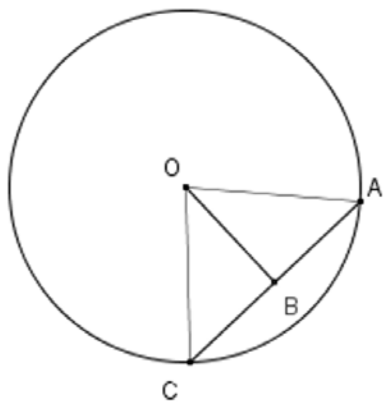
52. The curve $y = ax^3 + bx^2 - x + 5$ has a point of inflexion at (1, -2) Find the values of a and b.
53. Draw the gradient function of the curve below: 
54. Find all values of x for which the curve $f(x) = 2x^3 - 7x^2 - 5x + 4$ is concave downward.
55. Draw the gradient function of the curve below: 
56. Find the point of inflexion on the curve $y = x^3 - 6x^2 + 5x + 9$

57. What are the values of a and b?



58. The sum of the interior angles of a regular polygon is 2520°.
What is the size of each interior angle?

59. The circle below has centre O and OB bisects chord AC .

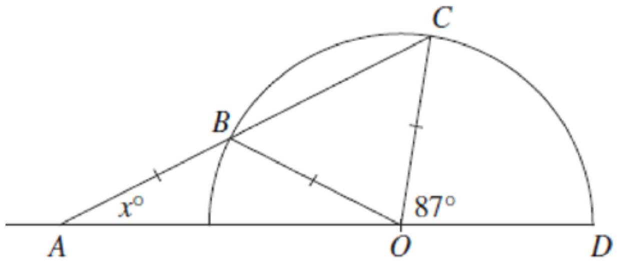


(i) Prove that ΔOAB is congruent to ΔOCB.

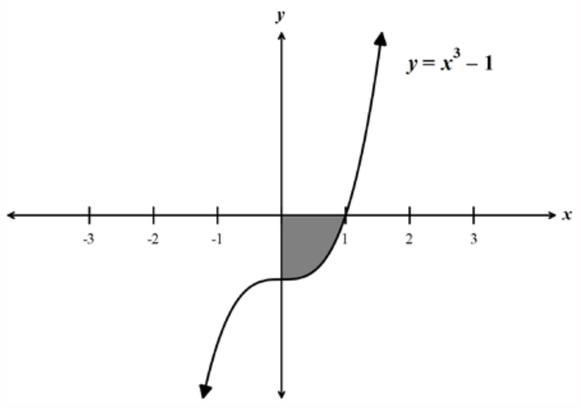
(ii) Prove that OB is perpendicular to AC.

60. The diagram shows a semicircle with centre O.
It is given that $AB = OB$, $\angle COD = 87°$ and $\angle BAO = x°$.

(i) Show that $\angle CBO = 2x°$ giving reasons.
(ii) Find the value of x, giving reasons.



61.



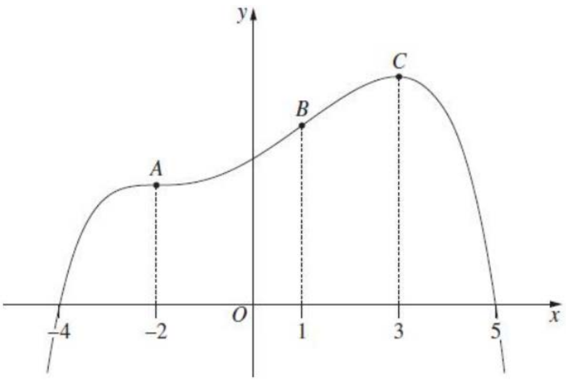
The region bounded by the $x - axis$, the $y - axis$ and the curve $y = x^3 - 1$ is rotated about the $y - axis$ to form a solid.

i) Find where the curve cuts the $y - axis$.

ii) Hence find the volume of the solid. (*leave your answer in exact form*)

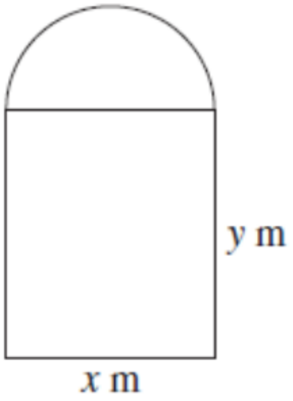
62. The diagram shows the graph of a function f(x). The graph has a horizontal point of inflexion at A, a point of inflexion at B and a maximum turning point at C.

Sketch the graph of the derivative f '(x).



63. Find the **exact** volume of the solid formed when the area under the curve $y = x^{\frac{3}{2}}$, above the x-axis and between $x = 1$ and $x = 2$ makes a revolution about the x-axis.

64. The diagram shows a window consisting of two sections.
The top section is a semicircle of diameter x m.
The bottom section is a rectangle of width x m and height y m.
The entire frame of the window, including the piece that separates the two sections, is made using 10 m of thin metal.
The semicircular section is made of coloured glass and the rectangular section is made of clear glass.
Under test conditions the amount of light coming through one square metre of the coloured glass is 1 unit and the amount of light coming through one square metre of the clear glass is 3 units.
The total amount of light coming through the window under test conditions is L units.

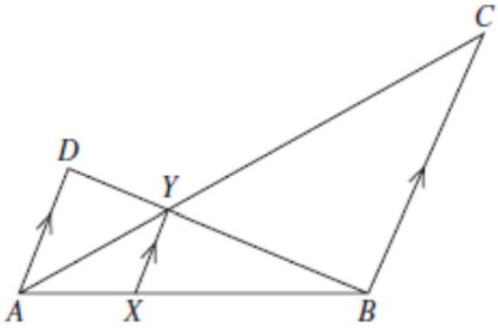


(i) Show that $y = 5 - x(1 + \frac{\pi}{4})$.

(ii) Show that $L = 15x - x^2(3 + \frac{5\pi}{8})$

(ii) Find the values of x and y that maximise the amount of light coming through the window under test conditions.

65. The diagram shows triangles ABC and ABD with AD parallel to BC.
The sides AC and BD intersect at Y.
The point X lies on AB such that XY is parallel to AD and BC.



(i) Prove that Δ ABC is similar to Δ AXY.

(ii) Hence, or otherwise, prove that $\frac{1}{XY} = \frac{1}{AD} + \frac{1}{BC}$