PICTON HIGH SCHOOL

Creating Opportunities Achieving Success

YEAR 12 Ext 1 Mathematics
Half-Yearly Examination/Sighted Test

Due Date: 18th March 2019 **Monday Week 8**	**Assessment Name: Half-Yearly Exam/Sighted Test**
Mark: /35	**Weighting: 30%** **Length: 60 minutes**

SYLLABUS OUTCOMES TO BE ASSESSED:

HE1 **Appreciates** the interrelationships between ideas drawn from different areas of mathematics
HE2 **Uses** inductive reasoning in the construction of proofs
HE3 **Uses** a variety of strategies to investigate mathematical models of situations involving binomial probability, projectiles, simple harmonic motion, or exponential growth and decay
HE4 **Uses** the relationship between functions, ~~inverse functions~~ and derivatives
HE6 **Determines** integrals by reduction to a standard form through a given substitution
HE7 **Evaluates** mathematical solutions to problems and communicates them in an appropriate form

DIRECTIVES TO BE ASSESSED:

Uses	To achieve a solution by using mathematical processes
Appreciates	To understand fully, grasp the full meaning of
Determines	To establish exactly by calculation
Evaluates	To find a numerical expression or equivalent for an equation, formula, or function.

TASK DESCRIPTION:

You will complete a 60 minute exam, with a five minutes reading time, covering the Ext 1Mathematics topics listed below. Note Preliminary outcomes can be tested as a part of these topics.
The exam will involve 5 multiple choice questions and 2 extended response questions of equal value.
A board approved formulae sheet will be provided with your test paper.

The exam will consist of -

Section 1 – Five multiple choice questions worth 5 marks (1 mark each)
Section 2 – Two extended response questions worth 30 marks (15 marks each)

The topics assessed are:

- Methods of Integration (11.5) (Harder applications and Substitution)
- Iterative methods for numerical estimation of the roots of a polynomial equation (16.4) (Halving the Interval and Newton's method)
- Locus and the Parabola(Parametric)
- Binomial Theorem
- Harder applications of Mathematics topics (which could include graphing, polynomials, differentiation and integration)

Equipment required:

- Board approved scientific calculator
- Pens, ruler, pencils

This task will be completed under exam conditions.

ASSESSMENT CRITERIA – STUDENT CHECKLIST:

- Have you revised these topics?
- Do you have all the equipment?
- Are you familiar with the formula sheet and the formula you may need to learn?
- Have you completed the sample tests from the Google classroom chhpshx for revision and requested assistance?

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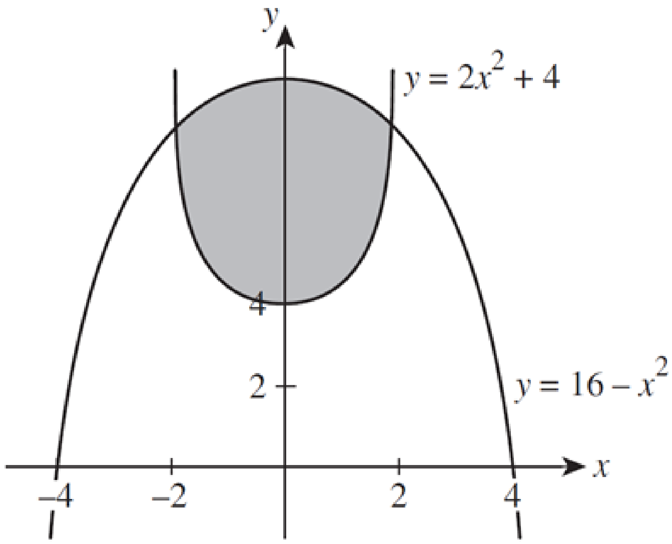
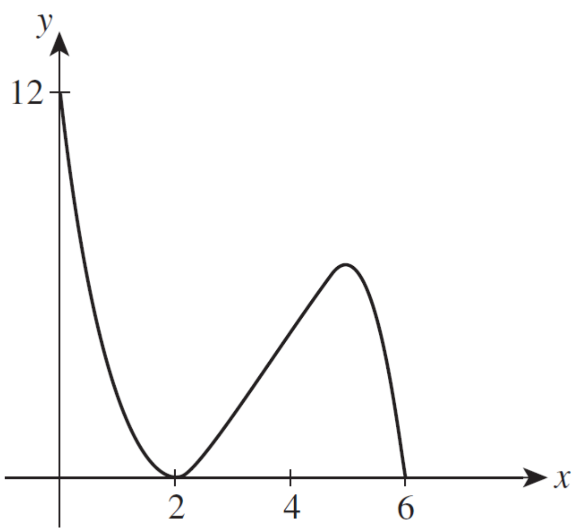
1. The diagram shows graphs with equations $y = 16 - x^2$ and $y = 2x^2 + 4$.



2.

The shaded area equals

The diagram shows the graph with equation $y = k(x + c)^2(x - 6)$.



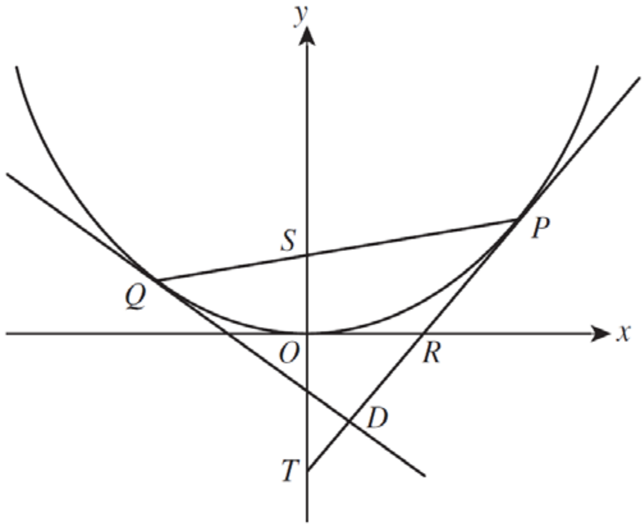
What are the values of c and k?

3. Use one step of Newton's method to find a closer approximation for the root of the equation $\cos(2x) - x = 0$ if the initial approximation is $x_1 = 1$. Give your answer correct to two decimal places.

4.

Using the substitution $u = 9 - x^3$, find the upper limit of integration b for the definite integral $\int_0^b x^2\sqrt{9 - x^3}\,dx = 5\frac{7}{9}$.

5. For the parabola, $x^2 = 16y$.



(i) If P is a point on the curve $x^2 = 16y$, where $x = 8p$, show that the value of the y-coordinate in terms of p is $4p^2$.

(ii) Find the equation of the tangent to $x^2 = 16y$ at $(8p, 4p^2)$.

(iii) The focal chord from P meets $x^2 = 16y$ again at Q.

Find the coordinates of Q in terms of p.

(iv) Show that the point of intersection, D, of the tangents from P and Q is on the directrix.

equation of the locus of points generated by the tangents at ?

6. Evaluate $\int_0^1 \frac{1}{\sqrt{2-x^2}}\,dx$.

7. Let $g(x) = 2x^3 + x + 4$.

(i) Show that $g(x) = 0$ has a root between the integers -1 and -2.

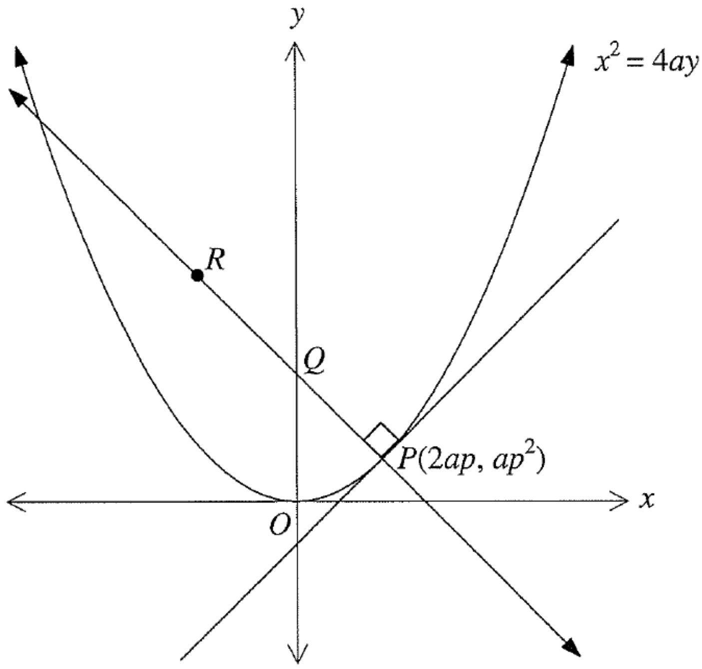
(ii) Taking $x = -1.5$ as the first approximation to this root, use one application of Newton's method to obtain a better approximation for this root.

Give this approximation correct to 2 significant figures.

(iii) Explain why the function $y = g(x)$ has only one x-intercept.

8. Find the term independent of x in the expansion of $\left(x^2 - \frac{1}{2x}\right)^{12}$.

9.



The diagram shows a variable point $P\left(2ap, ap^2\right)$ on the parabola $x^2 = 4ay$.

The normal to the parabola at *P* intersects the *y*-axis at *Q*. The point *Q* is the midpoint of *PR*.

The equation of the normal is $x + py - 2ap - ap^3 = 0$. (Do NOT prove this.)

(i) Find the coordinates of the point *Q*.

(ii) The locus of the point *R* is a parabola.

Find the equation of this parabola in Cartesian form and state its vertex.

10. In the binomial expansion of $\left(1+\frac{x}{k}\right)^n$, the coefficient of x^3 is twice the coefficient of x^2.

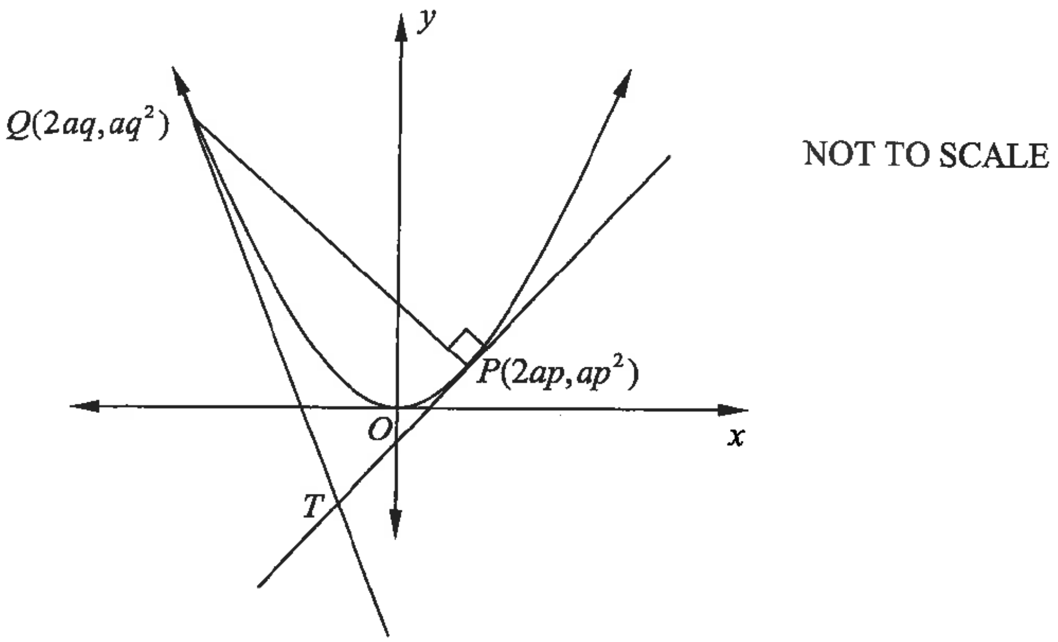
Prove that $n = 6k + 2$.

11. (i) Show that ${}^nC_k = {}^nC_{n-k}$.

(ii) Use the identity $(1+x)^n(1+x)^n \equiv (1+x)^{2n}$ to show that

$$\sum_{k=0}^{n}\left({}^nC_k\right)^2 = \frac{(2n)!}{(n!)^2}$$

12. The points $P\left(2ap, ap^2\right)$ and $Q\left(2aq, aq^2\right)$ lie on the parabola $x^2 = 4ay$. The equation of the tangent at the point P is $y = px - ap^2$ and the gradient of the chord PQ is $\frac{p+q}{2}$. The point T is the intersection of the tangents at P and Q.



(i) Show that the coordinates of T are $\left(a(p+q), apq\right)$.

(ii) The chord PQ is also the normal at P. Show that $p + q + \frac{2}{p} = 0$.

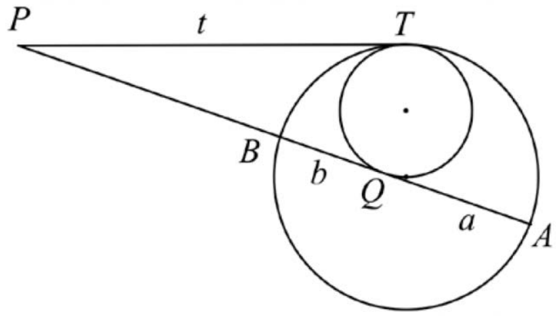
(iii) Hence, or otherwise, show that the equation of the locus of T as P moves on the parabola is $y = \frac{-4a^3}{x^2} - 2a$.

13. A parabola has parametric equations $x = \frac{t}{2}$ and $y = \frac{4t^2}{3}$.

What is the Cartesian equation of the parabola?

14. Solve $|x + 3| < |x - 3|$

15. Two circles have a common point T.
PT is a common tangent to the circles.
PA is the tangent to the smaller circle at Q.
$PT = t$, $QA = a$ and $QB = b$.
Which is an expression for t in terms of a and b.

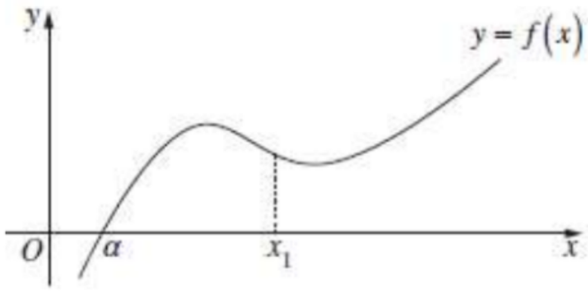


16. Find $\int (3x^2 - 4)(2x^3 - 4x^2 + 5)^4 \, dx$ using the substitution $u = 2x^3 - 4x^2 + 5$.

17. Find the coefficient of x^4 in the expansion of $\left(2x - \frac{1}{x}\right)^{10}$.

18. Expand and simplify $(x + y)^6$

19. The diagram shows the graph of a function $f(x)$. The equation $f(x) = 0$ has a root at $x = \alpha$. The value x_1, as shown in the diagram, is chosen as a first approximation of α. A second approximation, x_2, of α is obtained by applying Newton's method once, using x_1 as the first approximation. Using a diagram, or otherwise, explain why x_1 is a closer approximation than x_2.



20. Evaluate $\int_0^3 \frac{x}{\sqrt{x+1}} \, dx$, using the substitution $x = u^2 - 1$.

21. Which integral is obtained when the substitution $u = 1 + 2x$ is applied to $\int x\sqrt{1+2x} \, dx$?

(A) $\frac{1}{4}\int (u-1)\sqrt{u} \, du$ (B) $\frac{1}{2}\int (u-1)\sqrt{u} \, du$ (C) $\int (u-1)\sqrt{u} \, du$ (D) $2\int (u-1)\sqrt{u} \, du$

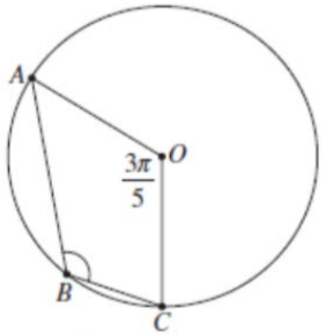
22. Use the substitution $u = x - 4$ to find $\int x\sqrt{x-4} \, dx$.

23. (i) Sketch the graph of $y = |2x - 1|$.
(ii) Hence, or otherwise, solve $|2x - 1| \le |x - 3|$.

24. The points A, B and C lie on a circle with centre O, as shown in the diagram.

The size of $\angle AOC$ is $\frac{3\pi}{5}$ radians.

What is the size of $\angle ABC$ in radians?



Not to scale

25. a) Show that that a root of $x^3 - 3x^2 - 9x + 1$ lies between $x = 4$ and $x = 5$

b) By halving the interval,show that the root lies between 4.75 and 4.875