



Year 11 Preliminary Mathematics Extension 1 2024

Task 1 -Sighted Assessment Notification

Due Date: Thursday 30 May	Assessment Name: Sighted Task
Marks: 40 (Hand in: 10, In class: 30)	Weighting: 30 %

SYLLABUS OUTCOMES TO BE ASSESSED:

ME11-1 uses algebraic and graphical techniques to **solve**, and where appropriate, **compare** alternative solutions to problems

MA11-2 provides reasoning to **support** conclusions which are appropriate to the context

ME11-6 uses appropriate technology to **investigate**, **organise** and **interpret** information to solve problems in a range of contexts

ME11-7 communicates making comprehensive use of mathematical language, notation, diagrams and graphs

DIRECTIVES TO BE ASSESSED:

Compare: show how things are similar or different

Solve: find the value of the unknown pronumeral in an equation or inequality

Use: to manipulate something for a particular purpose to solve mathematical problems

Provides: to give a suitable solution or justification in context of the mathematical problem

TASK DESCRIPTION:

You have been given a number of questions from which a 50-minute examination will be created. The examination will include **12 short answers** each varying from 1 to 3 marks.

You will be required to prepare for this examination by completing the attached questions as a form of study/revision. The examination questions will be taken directly from the attached questions. Some of the following questions may be slightly modified.

The topics that will be assessed are:

- Combinatorics (Permutations, Combinations and Binomial Expansions)
- Functions (Remainder Theorem, Factor Theorem, Sums and Products of Roots)

ASSESSMENT CRITERIA – STUDENT CHECKLIST:

- Have you completed the questions attached to this notification?
- Have you asked for additional help?
- Have you revised all content in the topics assessed?

Some of the following questions may be slightly modified. They may be used as multiple-choice questions or as find the error in the question.

Syllabus is available on

<https://www.educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017>

COMBINATORICS (Syllabus Ref: A1.1)

List and count the number of ways an event can occur

- Use the fundamental counting principle (also known as the multiplication principle)
- Use factorial notation to describe and determine the number of ways n different items can be arranged in a line or a circle
- Solve simple problems and prove results using the pigeonhole principle
- Understand and use permutations to solve problems
- Solve problems involving permutations and restrictions with or without repeated objects
- Understand and use combinations to solve problems
- Solve practical problems involving permutations and combinations, including those involving simple probability situations

BINOMIAL EXPANSIONS (Syllabus Ref: A1.2)

- Expand for small positive integers n
- Derive and use simple identities associated with Pascal's triangle

REMAINDER AND FACTOR THEOREMS (Syllabus Ref: F2.1)

- Define a general polynomial in one variable, x , of degree n with real coefficients to be the expression: $ax^n + \dots + a_0$, where $a_n \neq 0$
- Use division of polynomials to express $P(x)$ in the form $P(x) = A(x) \cdot Q(x) + R(x)$ where $\deg R(x) < \deg A(x)$ and $A(x)$ is a linear or quadratic divisor, $Q(x)$ the quotient and $R(x)$ the remainder
- Prove and apply the factor theorem and the remainder theorem for polynomials and hence solve simple polynomial equations

SUMS AND PRODUCTS OF ROOTS (Syllabus Ref: F2.2)

- Solve problems using the relationships between the roots and coefficients of quadratic, cubic and quartic equations
- Graph a variety of polynomials and investigate the link between the root of a polynomial equation and the zero on the graph of the related polynomial function

~~The following dot point will need to be taught after calculus is introduced.~~

- ~~Determine the multiplicity of a root of a polynomial equation~~

Question 1

In how many different ways can 4 people be chosen from a group of 20 people?

Question 2

Marty the Martian has an infinite number of red, blue, yellow and black socks in a drawer. If Marty is pulling out socks in the dark, what is the smallest number of socks that Marty must pull out of the drawer to guarantee getting ten socks of the same colour?

Question 3

How many numbers greater than 6000 can be formed with the digits 1, 4, 5, 7, 8 if no digit is repeated?

Question 4

Consider the word STATISTICS.

- (i) How many arrangements of the letters are there?
- (ii) How many arrangements of the letters are there where the A and C are next to each other?

Question 5

A school mathematics department consists of 5 male and 5 female teachers. How many different exam-writing committees comprising 3 teachers could be formed if there must be at least one teacher of each gender on the committee?

Question 6

Show that if 100 unique numbers are in a set, then at least 17 of them have the same remainder when it is divided by 6.

Question 7

An extended family sits down to dinner at a round table. The family consists of 5 adults and 2 children.

Find the number of arrangements possible:

- (i) if there are no restrictions placed on seating;
- (ii) if two of the adults must sit together;
- (iii) if neither of the children can be seated together.

Question 8

A set of 20 students is made up of 10 students from Year 11 and 10 students from Year 12. Five students are to be selected from the set of 20. The order of selection is unimportant.

- (i) What is the total number of possible selections?
- (ii) What is the total number of selections, if there are at least two Year 11 students and at least two Year 12 students in the group of five?

Question 9

A coin is biased such that the probability of a head is 0.8. The probability that exactly four tails will be observed when the coin is flipped ten times is:

- A. $10 \times 0.2^6 \times 0.8^4$ B. $\binom{10}{4} \times 0.2^4 \times 0.8^6$ C. $\binom{10}{4} \times 0.2^6 \times 0.8^4$ D. $10 \times 0.2^4 \times 0.8^6$

(show reasoning for your solution)

Question 10

What is the coefficient of the x^3 term in the expansion of $(x + 5)^4$?

Question 11

What is the coefficient of the x^3 term in the expansion of $(2 - 3x)^8$

Question 12

If $(1 - 3\sqrt{2})^6 = a + b\sqrt{2}$, use the binomial expansion to find the values of a and b .

Question 13

12 people need to be split up into teams for a quiz.

- (i) Explain why the number of ways of splitting them into two groups of the same size is $\frac{1}{2} \times {}^{12}C_6$
- (ii) How many ways are there of splitting them into two groups of any size (but there must be at least one person in each group)?

Question 14

The coefficient of x in the expansion of $(a + x)^6$ is 1280. What is the value of a ?

Question 15

Divide $P(x) = x^3 + 2x^2 - x + 3$ by $(x - 2)$ and write in the form $P(x) = (x - 2)Q(x) + R$, where R is the remainder.

Question 16

For the polynomial $2x^3 + x^2 - 7x - 6 = 0$.

- i) Show that $(x+1)$ is a factor.
- ii) Hence, factorise as products of its linear factors

Question 17

- i) Show $f(x) = x^4 - 7x^3 + 10x^2$ has zeros 2 and 5
- ii) Find the roots of $f(x) = 0$

Question 18

Find the values of a, b and c for which. $3x^2 - 4x + 5 \equiv a(x - 1)(x - 2) + b(x - 2) + c$

Question 19

The polynomial $P(x) = x^3 + ax + 12$ has a factor $(x + 3)$. Find the value of a .

Question 20

Find the quotient $Q(x)$, and the remainder, $R(x)$, when the polynomial $P(x) = 3x^3 - 7x^2 + 2x + 4$ is divided by $(x - 3)$

Question 21

A polynomial is given by $P(x) = x^3 + ax^2 + bx - 18$. Find the values of a and b if $(x + 2)$ is a factor of $P(x)$ and that $P(x)$ has a remainder of -24 when it is divided by $(x - 1)$.

Question 22

Find values of a and b if $f(x) = ax^3 + bx^2 - 2x + 5$ has a double root at $x = 1$

Question 23

If α , β and γ are the roots of $x^3 - 3x^2 + x - 5 = 0$, find

(i) $\alpha + \beta + \gamma$

(ii) $\alpha\beta\gamma$

(iii) $\alpha^2 + \beta^2 + \gamma^2$

(iv) $(\alpha + 1)(\beta + 1)(\gamma + 1)$

Question 24

One root of $2x^3 + 7x^2 - x + 1 = 0$ is 3. If α and β are the other two roots, find

(i) $\alpha + \beta$

(ii) $\alpha\beta$

Question 25

If two of the roots of $3x^3 + 11x^2 + 8x - 4 = 0$ are equal, find all the roots